International Journal of Mathematics and
Computer Applications Research (IJMCAR)
ISSN(P): 2249-6955 ISSN(E): 2249-8060
Vol. 4, Issue 5, Oct 2014, 15-28

INVENTORY MODEL FOR DETERIORATING ITEMS HAVING TWO COMPONENT MIXTURE OF PARETO LIFETIME AND TIME DEPENDENT DEMAND

VIJAYALAKSHMI. G[1], NIRUPAMA DEVI. K[2] & SRINIVASA RAO. K[3]

[1]Research Scholar, Department of Statistics, Andhra University, Visakhapatnam, Andhra Pradesh, India

[2,3]Professor, Department of Statistics, Andhra University, Visakhapatnam, Andhra Pradesh, India

ABSTRACT

Inventory management and control is a prerequisite for optimal utilization of resources in several organizations. Due to unpredicted lifetime of the commodity the inventory models for deteriorating items have gained lot of importance in several market yards, food processing industries. To have accurate analysis of inventory systems we develop and analyze an inventory model for deteriorating items having two component mixture of Pareto lifetime and time dependent demand. Here it is assumed that the procurement of items is done from two different sources and the lifetime of the commodity is heterogeneous having two different natures. The lifetime of the commodity is characterized by two component mixture of Pareto distribution. It is further assumed that the demand is a function of time and follows power pattern. The power pattern demand includes increasing, decreasing and constant rate of demand. With suitable cost considerations the total cost function is derived. By minimizing the total cost function the optimal ordering policies are derived. Through numerical illustration the sensitivity of the model with respect to the input parameters and costs is studied. It is observed that the lifetime distributional parameters have significant influence on the optimal ordering policies of the model. This model also includes some of the earlier models as particular cases.

KEYWORDS: Inventory Model, Deterioration, Power Pattern Demand, Two Component Mixture of Pareto Lifetime

1 INTRODUCTION

Inventory model is a mathematical representation of inventory system which is used for determining the optimal ordering policies for inventory management and control. Inventory models are extensively used for planning and scheduling several inventory systems arising at places like production processes, market yards, assembly lines, warehouses, etc. Inventory models are broadly classified into two categories based on the nature of commodity. They are (i) inventory models for deteriorating items and (ii) inventory models with infinite lifetime. The inventory models for deteriorating items are again divided into two groups viz., (i) inventory models for deteriorating items with fixed lifetime and (ii) inventory models for deteriorating items with random lifetime. Starting from the first inventory model for deteriorating items by Wihitin (31) much work has been reported regarding inventory problems dealing with deteriorating items. Raafat (20), Goyal and Giri (10), Ruxien Li, et al. (22), Pentico and Drake (18) have reviewed the inventory models for deteriorating items. Deterioration is usually defined as the damage, decay, spoilage, evaporation and obsolescence of item. In real life many items deteriorate due to inherent nature, for example fruits, vegetables, food items, Sea foods, agricultural products, textiles, chemicals, medicines, electronic components, cement, fertilizers, oils, gas etc., are some of the deteriorating items which are kept in inventory at various places. Recently much work has been reported in inventory models for deteriorating items. In developing the inventory model it is needed to ascribe a probability distribution to the

lifetime of the commodity. Ghare and Schader (8), Shah and Jaiswal (23), Cochen (5), Aggarwal (2), Dave and Shah (7), Pal (17), Kalpakam and Sapna (13), Giri and Chaudhuri (9) assumed that the lifetime of commodity follows an exponential distribution. Tadikamalla (27) assumed Gamma distdribution to the lifetime of the commodity. Covert and Philip (6), Philip (19), Goel and Aggarwal (1), Venkata Subbaiah et al. (30), assumed Weibull distribution to the lifetime of the commodity, Nirupama Devi et al. (15) developed inventory model with mixture of Weibull distribution for the lifetime of the commodity, Srinivasa Rao et al. (26) developed an inventory model with generalized Pareto lifetime. Xu and Li (32) developed a two ware house inventory model for deteriorating items with time dependent demand. Rong et al. (21) studied a two ware house inventory model for deteriorating items with partially/fully backlogged shortages and fuzzy lead time. Srinivasa Rao K et al. (24) developed an inventory model for deteriorating items having additive exponential lifetime and selling price dependent demand rate. Chang and Lin (11) studied an inventory model for deteriorating items with stock dependent demand. Biswajit Sarkar (4) developed an EOQ Model with delay in payments and stock dependent demand in presence of imperfect production. Vinod Kumar Mishra et al. (29) have developed an inventory model for deteriorating items with time dependent demand and time varying holding cost under partial backlogging. Khanra et al. (14) have studied an inventory model with time dependent demand and shortages under trade credit policy. Jayjayanthi Ray et al. (12) have developed an inflationary inventory model with stock dependent demand and shortages. Bhanu Priya Dash et al. (3) have developed an inventory model for deteriorating items with exponential declining demand and time varying holding cost.

In all these models they assumed that the lifetime of the commodity is random and follows a probability distribution having the homogenous nature. However in some situations prevailing at places like fruit and vegetable markets, food processing industries, photo chemicals and pharmaceutical industries the stock on hand is procured from various sources for the same type of items. Even though the nature of the items does not differ there will be some inherent variation due to the source from which they are procured. So the efficiency of the inventory model heavily depends upon the probability distribution ascribed to the lifetime of the commodity under consideration. In general the stock on hand with respect to their perishability can be considered as heterogeneous population consisting of two types of lifetimes viz, shorter and longer lifetimes. When the items are mixed in stock, it is difficult to isolate each item with respect to their lifetime of the commodity. So in order to associate suitable probability model one has to consider mixture of distributions. Very little work has been done reported regarding inventory model for deteriorating items having heterogeneous lifetime except the works of Srinivasa Rao et al. (25) and Nirupama Devi et al. (16) who have studied the inventory model with the assumption that the lifetime of the commodity follows a two component mixture of Weibull distribution. But Weibull distribution has some draw backs such as having three parameters for location, shape and scale. If the shape parameter is large then the distribution will be right skewed and having long lower tail, which may not coincide with the lifetime characteristic of the commodity under_study. Hence in this paper we develop and analyze an inventory model for deteriorating items with the assumption that the lifetime of the commodity is random and follows a two component mixture of Pareto distribution. The Pareto distribution is capable of portraying left skewed distribution with long upper tail which matches with the lifetime of the commodity such as food grains, sea foods, and chemicals. The two component mixture of Pareto distribution also includes the Pareto distribution as a particular case when the mixing parameter becomes zero. The rest of the paper is organized as follows: Section 2 describes the assumptions and notation used throughout the paper. In section 3, we formulate a mathematical model as a cost minimization problem for with shortages. In section 4, we formulate a mathematical model as a cost minimization problem for without shortages.

In section 5, a numerical example is given to illustrate the model. Sensitivity analysis for various parameters is taken in section 6. In section 7, conclusions of the work and suggestions for future research are given.

2 Inventory Model for Deteriorating Items Having Two Component Mixture of Pareto Lifetime and Demand as a Function of Time

Consider an inventory system for deteriorating items in which the life time of the commodity is random and follows two component mixture of Pareto distribution with probability density function of the form

$$f(t) = \alpha\beta_1\theta^{\beta_1}t^{-(\beta_1+1)} + (1-\alpha)\beta_2\theta^{\beta_2}t^{-(\beta_2+1)} \qquad t \geq \theta$$

The distribution function of t is $F(t) = \alpha\left[1-\left(\frac{\theta}{t}\right)^{\beta_1}\right] + (1-\alpha)\left[1-\left(\frac{\theta}{t}\right)^{\beta_2}\right]$

The mean lifetime of the commodity is

$$\mu = \alpha\frac{\beta_1\theta}{(\beta_1-1)} + (1-\alpha)\frac{\beta_2\theta}{(\beta_2-1)} \qquad t \geq \theta,\ \beta_1 > 1,\ \beta_2 > 1$$

The variance of lifetime of the commodity is

$$Var(t) = \alpha\frac{\beta_1\theta^2}{\beta_1-2} + (1-\alpha)\frac{\beta_2\theta^2}{\beta_2-2} - \left[\alpha\frac{\beta_1\theta}{(\beta_1-1)} + (1-\alpha)\frac{\beta_2\theta}{(\beta_2-1)}\right]^2 \qquad t \geq \theta,\ \beta_1 > 2,\ \beta_2 > 2$$

The instantaneous rate of deterioration is

$$h(t) = \frac{f(t)}{1-F(t)} = \frac{\alpha\beta_1\theta^{\beta_1}t^{-(\beta_1+1)} + (1-\alpha)\beta_2\theta^{\beta_2}t^{-(\beta_2+1)}}{1-\alpha\left[1-\left(\frac{\theta}{t}\right)^{\beta_1}\right] - (1-\alpha)\left[1-\left(\frac{\theta}{t}\right)^{\beta_2}\right]} \tag{1}$$

ASSUMPTIONS

The demand rate is a power function of time, Lead time is zero,

Cycle length is T, Shortages are allowed and fully back logged.

NOTATION

Q: Ordering quantity in one cycle

h: Inventory holding cost per unit per unit time

A: Ordering cost

π: Shortage cost per unit per unit time

C: Cost per unit

r: Total demand during the cycle

n: Pattern index

3 INVENTORY MODEL FOR DETERIORATING ITEMS WITH TIME DEPENDENT DEMAND WITH SHORTAGES

Consider an inventory system for deteriorating items for which the lifetime of commodity is random and follows two component mixture of Pareto distribution with the probability density function of the form as given in section 2.

In this model the stock level (initial inventory) is Q at time $t = 0$. The stock decreases during the period $(0, \theta)$ due

to demand and due to demand and deterioration during the period (θ, t_1). At time t_1 inventory reaches zero and back orders accumulate during the period (t_1, T).

Let $I(t)$ be the inventory level of the system at time t $(0 \le t \le T)$

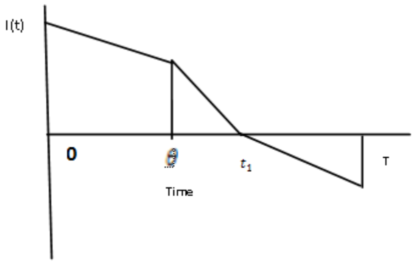


Figure 1: Schematic Diagram Representing the Inventory Level

The differential equations governing the instantaneous state of $I(t)$ over the cycle of length T are

$$\frac{d}{dt} I(t) = -\frac{r t^{\frac{1}{n}-1}}{n T^{\frac{1}{n}}} \qquad 0 \le t \le \theta \tag{2}$$

$$\frac{d}{dt} I(t) + h(t) I(t) = -\frac{r t^{\frac{1}{n}-1}}{n T^{\frac{1}{n}}} \qquad \theta \le t \le t_1 \tag{3}$$

$$\frac{d}{dt} I(t) = -\frac{r t^{\frac{1}{n}-1}}{n T^{\frac{1}{n}}} \qquad t_1 \le t \le T \tag{4}$$

where, $h(t)$ is as given in equation (1), with the initial conditions $I(0) = Q$, $I(t_1) = 0$. Substituting $h(t)$ given in equation (1) in equations (2), (3), and (4) and solving the above differential equations, the on hand inventory at time t can be obtained as

$$I(t) = \frac{r}{T^{\frac{1}{n}}}\left\{\theta^{\frac{1}{n}} - t^{\frac{1}{n}} + \frac{1}{n}\int_{\theta}^{t_1}\left[1-\alpha\left(1-\left(\frac{\theta}{u}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{u}\right)^{\beta_2}\right)\right]^{-1} u^{\frac{1}{n}-1} du\right\} \qquad 0 \le t \le \theta \tag{5}$$

$$I(t) = \frac{r}{n T^{\frac{1}{n}}}\left[1-\alpha\left(1-\left(\frac{\theta}{t}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{t}\right)^{\beta_2}\right)\right]$$

$$\int_{t}^{t_1}\left[1-\alpha\left(1-\left(\frac{\theta}{u}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{u}\right)^{\beta_2}\right)\right]^{-1} u^{\frac{1}{n}-1} du \qquad \theta \le t \le t_1 \tag{6}$$

$$I(t) = \frac{r}{T^{\frac{1}{n}}}\left(t_1^{\frac{1}{n}} - t^{\frac{1}{n}}\right) \qquad t_1 \le t \le T \tag{7}$$

The Stock loss due to deterioration in the interval $(0, t)$ is

$$L(t) = I(0) - \frac{r {t_1}^{\frac{1}{n}}}{T^{\frac{1}{n}}} - I(t)$$

$$= \frac{r}{T^{\frac{1}{n}}}\left\{\theta^{\frac{1}{n}} - \frac{t^{\frac{1}{n}}}{n} + \frac{1}{n}\int_{\theta}^{t_1}\left[1-\alpha\left(1-\left(\frac{\theta}{u}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{u}\right)^{\beta_2}\right)\right]^{-1} u^{\frac{1}{n}-1}du\right.$$

$$\left. -\frac{1}{n}\left[1-\alpha\left(1-\left(\frac{\theta}{t}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{t}\right)^{\beta_2}\right)\right]\int_{t}^{t_1}\left[1-\alpha\left(1-\left(\frac{\theta}{u}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{u}\right)^{\beta_2}\right)\right]^{-1} u^{\frac{1}{n}-1}du\right\}$$

$$\theta \le t \le t_1 \qquad (8)$$

The ordering quantity, Q in the cycle of length T is

$$Q = \frac{r}{T^{\frac{1}{n}}}\left\{\theta^{\frac{1}{n}} + \frac{1}{n}\int_{\theta}^{t_1}\left[1-\alpha\left(1-\left(\frac{\theta}{t}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{t}\right)^{\beta_2}\right)\right]^{-1} t^{\frac{1}{n}-1}dt + \frac{t_1^{\frac{1}{n}-1}}{n}(T-t_1)\right\} \qquad (9)$$

Let $K(t_1,T)$ be the total cost per unit time. Since the total cost is sum of the set up cost, cost of the units, the inventory holding cost, the shortage cost, $K(t_1,T)$ becomes

$$K(t_1,T) = \frac{A}{T} + \frac{CQ}{T} + \frac{h}{T}\left[\int_0^{\theta} I(t)\,dt + \int_{\theta}^{t_1} I(t)\,dt\right] + \frac{\pi}{T}\int_{t_1}^{T} -I(t)\,dt \qquad (10)$$

Substituting the values of $I(t)$ and Q given in equations (5), (6), (7) and (9) in equation (10), we obtain $K(t_1,T)$ as

$$K(t_1,T) = \frac{A}{T} + \frac{C}{T}\left\{\theta^{\frac{1}{n}} + \frac{1}{n}\int_{\theta}^{t_1}\left[1-\alpha\left(1-\left(\frac{\theta}{u}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{u}\right)^{\beta_2}\right)\right]^{-1} u^{\frac{1}{n}-1}du + \frac{t_1^{\frac{1}{n}-1}}{n}(T-t_1)\right\}$$

$$+ \frac{h}{T}\int_0^{\theta}\frac{r}{T^{\frac{1}{n}}}\left\{\theta^{\frac{1}{n}} - t^{\frac{1}{n}} + \frac{1}{n}\int_{\theta}^{t_1}\left[1-\alpha\left(1-\left(\frac{\theta}{u}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{u}\right)^{\beta_2}\right)\right]^{-1} u^{\frac{1}{n}-1}du\right\}dt$$

$$+ \frac{h}{T}\int_{\theta}^{t_1}\frac{r}{nT^{\frac{1}{n}}}\left\{\left[1-\alpha\left(1-\left(\frac{\theta}{t}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{t}\right)^{\beta_2}\right)\right]\right.$$

$$\left.\int_{t}^{t_1}\left[1-\alpha\left(1-\left(\frac{\theta}{u}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{u}\right)^{\beta_2}\right)\right]^{-1} u^{\frac{1}{n}-1}du\right\}dt - \frac{\pi}{T}\int_{t_1}^{T}\frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\left(t_1^{\frac{1}{n}} - t^{\frac{1}{n}}\right)dt \qquad (11)$$

3.1 Optimal Ordering Policies of the Model

In this section we obtain the optimal policies of the inventory system developed in section 3. The conditions for obtaining optimality are

$$\frac{\partial}{\partial t_1}K(t_1,T) = 0; \quad \frac{\partial}{\partial T}K(t_1,T) = 0 \quad and$$

$$Determinant\ of\ Hessian\ matrix\ D = \begin{vmatrix} \frac{\partial^2}{\partial t_1^2}K(t_1,T) & \frac{\partial}{\partial t_1\partial T}K(t_1,T) \\ \frac{\partial}{\partial T\partial t_1}K(t_1,T) & \frac{\partial^2}{\partial T^2}K(t_1,T)\end{vmatrix} > 0$$

Differentiating (11) with respect to t_1 and equating to zero, we get

$$C\left\{\left[1-\alpha\left(1-\left(\frac{\theta}{t_1}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{t_1}\right)^{\beta_2}\right)\right]^{-1}+\frac{n-1}{n}\frac{T}{t_1}-1\right\}+h\theta\left[1-\alpha\left(1-\left(\frac{\theta}{t_1}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{t_1}\right)^{\beta_1}\right)\right]^{-1}$$

$$+h\left[1-\alpha\left(1-\left(\frac{\theta}{t_1}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{t_1}\right)^{\beta_2}\right)\right]^{-1}\int_\theta^{t_1}\left[1-\alpha\left(1-\left(\frac{\theta}{t}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{t}\right)^{\beta_2}\right)\right]dt-\pi\left(T^{\frac{1}{n}}-t_1^{\frac{1}{n}}\right)=0 \quad (12)$$

Differentiating (11) with respect to T and equating to zero, we get

$$\frac{nAT^{\frac{1}{n}}}{(n+1)r}+C\left\{\theta^{\frac{1}{n}}+\frac{1}{n}\int_\theta^{t_1}\left[1-\alpha\left(1-\left(\frac{\theta}{u}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{u}\right)^{\beta_2}\right)\right]^{-1}u^{\frac{1}{n}-1}du\right\}$$

$$\frac{C}{(n+1)}\left[Tt_1^{\frac{1}{n}-1}-\left(\frac{n+1}{n}\right)Tt_1^{\frac{1}{n}-1}\right]+h\int_0^\theta\frac{r}{T^{\frac{1}{n}}}\left\{\theta^{\frac{1}{n}}-t^{\frac{1}{n}}+\frac{1}{n}\int_\theta^{t_1}\left[1-\alpha\left(1-\left(\frac{\theta}{u}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{u}\right)^{\beta_2}\right)\right]^{-1}u^{\frac{1}{n}-1}du\right\}dt$$

$$+\frac{h}{n}\int_\theta^{t_1}\left\{\left[1-\alpha\left(1-\left(\frac{\theta}{t}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{t}\right)^{\beta_2}\right)\right]\int_t^{t_1}\left[1-\alpha\left(1-\left(\frac{\theta}{u}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{u}\right)^{\beta_2}\right)\right]^{-1}u^{\frac{1}{n}-1}du\right\}dt$$

$$+\pi\left(T^{\frac{1}{n}}-t_1^{\frac{1}{n}}\right)\left[\frac{2T^{\frac{1}{n}}}{n+1}-\left(T^{\frac{1}{n}}-t_1^{\frac{1}{n}}\right)\right]=0 \quad (13)$$

Solving equations (12) and (13), we obtain the time at which shortages occur t_1^*, the optimal cycle length, T^* of T. The optimal ordering quantity, Q^* is obtained by substituting the optimal values of t_1, and T in equation (9), we get

$$Q^*=\frac{r}{T^{*\frac{1}{n}}}\left\{\theta^{\frac{1}{n}}+\frac{1}{n}\int_\theta^{t_1^*}\left[1-\alpha\left(1-\left(\frac{\theta}{t}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{u}\right)^{\beta_2}\right)\right]^{-1}t^{\frac{1}{n}-1}dt+\frac{t_1^{*\frac{1}{n}-1}}{n}(T^*-t_1^*)\right\} \quad (14)$$

4 INVENTORY MODEL FOR DETERIORATING ITEMS WITH TIME DEPENDENT DEMAND WITHOUT SHORTAGES

In section 3, the inventory model for deteriorating items with lifetime of commodity is random and follows two component mixture of Pareto distribution and with power pattern demand which is a function of time and with shortages is discussed. In this section the model without shortages is developed and analyzed. For developing the model, we assume that $\pi\to\infty$ and $t_1\to T$.

In this model the stock level (initial inventory) is Q at time $t=0$. The stock level decreases during the period $(0,\theta)$ due to demand and due to deterioration and demand durinsg the period $(\theta,\ T)$. At time T inventory reaches zero. Let $I(t)$ be the inventory level of the system at time t.

The differential equations governing the instantaneous state of $I(t)$ over the cycle of length T are

$$\frac{d}{dt}I(t)=-\frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}} \qquad 0\le t\le\theta \qquad (15)$$

$$\frac{d}{dt}I(t) + h(t)I(t) = -\frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}} \qquad \theta \le t \le T \tag{16}$$

Where, $h(t)$ is as given in equation (1), with the initial conditions $I(T) = 0$.

Substituting $h(t)$ given in equation (1) in equations (15) and (16) and solving the above differential equations, the on hand inventory at time t can be obtained as

$$I(t) = \frac{r}{T^{\frac{1}{n}}}\left\{\theta^{\frac{1}{n}} - t^{\frac{1}{n}} + \frac{1}{n}\int_{\theta}^{T}\left[1-\alpha\left(1-\left(\frac{\theta}{u}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{u}\right)^{\beta_2}\right)\right]^{-1} u^{\frac{1}{n}-1}du\right\} \qquad 0 \le t \le \theta \tag{17}$$

$$I(t) = \frac{r}{nT^{\frac{1}{n}}}\left[1-\alpha\left(1-\left(\frac{\theta}{t}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{t}\right)^{\beta_2}\right)\right]$$

$$\int_{t}^{T}\left[1-\alpha\left(1-\left(\frac{\theta}{u}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{u}\right)^{\beta_2}\right)\right]^{-1} u^{\frac{1}{n}-1}du \qquad \theta \le t \le T \tag{18}$$

The Stock loss due to deterioration in the interval $(0, t)$ is

$$L(t) = I(0) - \frac{rt^{\frac{1}{n}}}{T^{\frac{1}{n}}} - I(t) \qquad \theta \le t \le T$$

$$= \frac{r}{T^{\frac{1}{n}}}\left\{\left(\theta^{\frac{1}{n}} - t^{\frac{1}{n}}\right) + \frac{1}{n}\int_{\theta}^{T}\left[1-\alpha\left(1-\left(\frac{\theta}{u}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{u}\right)^{\beta_2}\right)\right]^{-1} u^{\frac{1}{n}-1}du\right.$$

$$-\frac{1}{n}\left[1-\alpha\left(1-\left(\frac{\theta}{t}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{t}\right)^{\beta_2}\right)\right]$$

$$\left.\int_{t}^{T}\left[1-\alpha\left(1-\left(\frac{\theta}{u}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{u}\right)^{\beta_2}\right)\right]^{-1} u^{\frac{1}{n}-1}du\right\} \qquad \theta \le t \le T \tag{19}$$

The ordering quantity, Q in the cycle length T is

$$Q = I(0) = \frac{r}{T^{\frac{1}{n}}}\left\{\theta^{\frac{1}{n}} + \frac{1}{n}\int_{\theta}^{T}\left[1-\alpha\left(1-\left(\frac{\theta}{t}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{t}\right)^{\beta_2}\right)\right]^{-1} t^{\frac{1}{n}-1}dt\right\} \tag{20}$$

Let $K(T)$ be the total cost per unit time. Since the total cost is sum of the set up cost, cost of the units, the inventory holding cost, the shortage cost, $K(T)$ becomes

$$K(T) = \frac{A}{T} + \frac{CQ}{T} + \frac{h}{T}\left[\int_{0}^{\theta} I(t)dt + \int_{\theta}^{T} I(t)dt\right] \tag{21}$$

Substituting the values $I(t)$ and Q given in equations (17),(18) and (20) in equation (21), we obtain

$$K(T) = \frac{A}{T} + \frac{Cr}{T^{\frac{1}{n}+1}}\left\{\theta^{\frac{1}{n}} + \frac{1}{n}\int_{\theta}^{T}\left[1-\alpha\left(1-\left(\frac{\theta}{u}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{u}\right)^{\beta_2}\right)\right]^{-1} u^{\frac{1}{n}-1}du\right\}$$

$$+\frac{hr}{T^{\frac{1}{n}+1}}\int_0^{\theta}\left\{\theta^{\frac{1}{n}}-t^{\frac{1}{n}}+\frac{1}{n}\int_{\theta}^{T}\left[1-\alpha\left(1-\left(\frac{\theta}{u}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{u}\right)^{\beta_2}\right)\right]^{-1}u^{\frac{1}{n}-1}du\right\}dt$$

$$+\frac{hr}{nT^{\frac{1}{n}+1}}\int_{\theta}^{T}\left\{\left[1-\alpha\left(1-\left(\frac{\theta}{t}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{t}\right)^{\beta_2}\right)\right]\right.$$

$$\left.\int_t^T\left[1-\alpha\left(1-\left(\frac{\theta}{u}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{u}\right)^{\beta_2}\right)\right]^{-1}u^{\frac{1}{n}-1}du\right\}dt \tag{22}$$

4.1 Optimal Ordering Policies of the Model

In this section, we obtain the optimal policies of the inventory system developed in section 4. To find the optimal values of T, we equate the first order derivative of $K(T)$ given in equation (22) with respect to T to zero. The conditions for obtaining optimality are

$$\frac{d}{dT}K(T)=0; \qquad \frac{d^2}{dT^2}K(T)>0;$$

$$\frac{d}{dT}K(T)=0 \ implies$$

$$\frac{nAT^{\frac{1}{n}}}{(n+1)r}+C\theta^{\frac{1}{n}}+C\int_{\theta}^{T}\left[1-\alpha\left(1-\left(\frac{\theta}{u}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{u}\right)^{\beta_2}\right)\right]^{-1}u^{\frac{1}{n}-1}du$$

$$+\frac{T^{\frac{1}{n}+1}}{(n+1)}\left[1-\alpha\left(1-\left(\frac{\theta}{T}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{T}\right)^{\beta_2}\right)\right]^{-1}$$

$$+h\int_0^{\theta}\left\{\theta^{\frac{1}{n}}-t^{\frac{1}{n}}+\frac{1}{n}\int_{\theta}^{T}\left[1-\alpha\left(1-\left(\frac{\theta}{u}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{u}\right)^{\beta_2}\right)\right]^{-1}du\right.$$

$$\left.-\frac{T^{\frac{1}{n}+1}}{(n+1)}\left[1-\alpha\left(1-\left(\frac{\theta}{T}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{T}\right)^{\beta_2}\right)\right]^{-1}\right\}dt$$

$$+h\int_{\theta}^{T}\left\{\left[1-\alpha\left(1-\left(\frac{\theta}{t}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{t}\right)^{\beta_2}\right)\right]\int_t^T\left[1-\alpha\left(1-\left(\frac{\theta}{u}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{u}\right)^{\beta_2}\right)\right]^{-1}u^{\frac{1}{n}-1}du\right.$$

$$\left.-\frac{T^{\frac{1}{n}+1}}{(n+1)}\left[1-\alpha\left(1-\left(\frac{\theta}{T}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{T}\right)^{\beta_2}\right)\right]^{-1}\left[1-\alpha\left(1-\left(\frac{\theta}{t}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{t}\right)^{\beta_2}\right)\right]\right\}dt=0 \tag{23}$$

Solving equation (23), we obtain the optimal cycle length, T^* of T. By Substituting the optimal value of T in equation (20), we get

$$Q^*=\frac{r}{T^{*\frac{1}{n}}}\left\{\theta^{\frac{1}{r}}+\frac{1}{n}\int_{\theta}^{T^*}\left[1-\alpha\left(1-\left(\frac{\theta}{t}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{t}\right)^{\beta_2}\right)\right]^{-1}t^{\frac{1}{n}-1}dt\right\} \tag{24}$$

5 NUMERICAL ILLUSTRATION

To illustrate the inventory model developed in section 3, section 4, we consider the inventory system with the following hypothetical values, we consider A = Rs.100, C = Rs.5, h = Rs.0.1, π = Rs.0.5, θ = 25, β_1 = 2, β_2 = 2.5, α = 0.4, n = 2, r = 100. By Using Math Cad 15 software we obtain the time at which shortages occur t_1, optimal values of cycle length T, ordering quantity Q, as 41.3180, 98.8056, 113.2300~113 units for section 3, optimal values of cycle length T, ordering quantity Q, as 40.6927, 119.0099~119 units for section 4.

6 SENSITIVITY ANALYSIS

The sensitivity analysis is carried to explore the effect of changes in model parameters and costs on the optimal policies, by varying each parameter (-15%, -10%, -5%, 0%, 5%, 10%, 15%) at a time for the model under study for sections 3, 4. The results are presented in **Tables 1, 2.** The relationship between the parameters and optimal cycle length T^*, and optimal ordering quantity Q^* is shown in **Figures 2, 3.**

Table 1: Sensitivity Analysis for with Shortages

Variation Parameter	Optimal Values	% Change In Parameter						
		-15	-10	-5	0	5	10	15
A	t_1^*	41.0111	41.1138	41.2161	41.3180	41.4193	41.5202	41.6206
	T^*	96.7722	97.4485	98.1264	98.8056	99.4862	100.1681	100.8513
	K^*	13.6486	13.7001	13.7513	13.8020	13.8525	13.9026	13.9523
	Q^*	112.8516	112.9781	113.1043	113.2300	113.3553	113.4802	113.6047
C	t_1^*	40.3218	40.6611	40.9930	41.3180	41.6366	41.9493	42.2565
	T^*	89.3784	92.4635	95.6057	98.8056	102.0642	105.3819	108.7595
	K^*	12.9024	13.2115	13.5112	13.8020	14.0843	14.3584	14.6247
	Q^*	112.2066	112.5586	112.8995	113.2300	113.5507	113.8621	114.1648
h	t_1^*	42.1203	41.8376	41.5706	41.3180	41.0782	40.8502	40.6330
	T^*	98.4278	98.5349	98.6616	98.8056	98.9652	99.1387	99.3247
	K^*	13.5598	13.6427	13.7234	13.8020	13.8787	13.9536	14.0268
	Q^*	114.6461	114.1398	113.6691	113.2300	112.8193	112.4341	112.0720
π	t_1^*	41.9310	41.7069	41.5035	41.3180	41.1482	40.9922	40.8484
	T^*	113.7680	108.0797	103.1364	98.8056	94.9837	91.5885	88.5545
	K^*	12.8902	13.2145	13.5178	13.8020	14.0690	14.3201	14.5568
	Q^*	113.3115	113.2841	113.2569	113.2300	113.2036	113.1780	113.1530
θ	t_1^*	36.3378	38.0101	39.6698	41.3180	42.9552	44.5823	46.1997
	T^*	96.1372	96.9207	97.8162	98.8056	99.8744	101.0107	102.2047
	K^*	14.9204	14.5222	14.1503	13.8020	13.4754	13.1683	12.8791
	Q^*	114.7308	114.1953	113.6963	113.2300	112.7932	112.3831	111.9972
β_1	t_1^*	42.5798	42.1232	41.7041	41.3180	40.9608	40.6292	40.3206
	T^*	100.9758	100.1863	99.4659	98.8056	98.1982	97.6374	97.118
	K^*	13.6683	13.7156	13.7601	13.8020	13.8417	13.8792	13.9148
	Q^*	113.9237	113.6729	113.4424	113.2300	113.0335	112.8513	112.6819
β_2	t_1^*	43.1757	42.4954	41.8792	41.3180	40.8043	40.3322	39.8965
	T^*	101.8050	100.7044	99.7097	98.8056	97.9800	97.2226	96.5251
	K^*	13.5902	13.6660	13.7364	13.8020	13.8634	13.9210	13.9752
	Q^*	114.0957	113.7834	113.4959	113.2300	112.9833	112.7537	112.5394
α	t_1^*	41.0746	41.1554	41.2365	41.3180	41.3998	41.4821	41.5647
	T^*	98.4007	98.5353	98.6702	98.8056	98.9415	99.0777	99.2143
	K^*	13.8298	13.8206	13.8113	13.8020	13.7927	13.7834	13.7740
	Q^*	113.1032	113.1455	113.1878	113.2300	113.2721	113.3141	113.3560
n	t_1^*	40.4303	40.7290	41.0252	41.3180	41.6066	41.8907	42.1700

Variation Parameter	Optimal Values	% Change In Parameter						
		-15	-10	-5	0	5	10	15
	T^*	93.0336	94.9388	96.8651	98.8056	100.7557	102.7117	104.6709
	K^*	14.5880	14.3139	14.0521	13.8020	13.5630	13.3342	13.1150
	Q^*	113.1092	113.1537	113.1940	113.2300	113.2617	113.2893	113.3130
r	t_1^*	41.6736	41.5425	41.4246	41.3180	41.2210	41.1325	41.0513
	T^*	101.2136	100.3198	99.5221	98.8056	98.1587	97.5717	97.0366
	K^*	11.8817	12.5223	13.1624	13.8020	14.4414	15.0804	15.7192
	Q^*	96.6198	102.1571	107.6938	113.2300	118.7658	124.3012	129.8363

Following inferences can be made from **Table 1.** As the ordering cost A increases the optimal time at which shortages occur t_1^*, the optimal cycle length T^* and optimal quantity Q^*, are increasing. As the cost per unit C increases, the optimal time at which shortages occur t_1^*, the optimal cycle length T^*, and optimal ordering quantity Q^* are increasing. As the holding cost h increases the optimal time at which shortages occur t_1^*, and optimal ordering quantity Q^* are decreasing and the optimal cycle length T^*, is increasing. As the shortage cost π increases, the optimal time at which shortages occur t_1^*, and optimal ordering quantity Q^*, and optimal cycle length T^*, are decreasing. As the scale parameter θ increases, the optimal time at which shortages occur t_1^*, the optimal cycle length T^*, are increasing and optimal ordering quantity Q^*, is decreasing. As the shape parameter β_1 increases, optimal time at which shortages occur t_1^*, the optimal cycle length T^*, optimal ordering quantity Q^* are decreases.

As the shape parameter β_2 increases, optimal time at which shortages occur t_1^*, the optimal cycle length T^*, optimal ordering quantity Q^* are decreases. As the mixing parameter □ increases, optimal time at which shortages occur t_1^*, the optimal cycle length T^*, optimal ordering quantity Q^* are increasing. As the pattern index parameter n increases, the optimal time at which shortages occur t_1^*, the optimal cycle length T^*, and optimal ordering quantity Q^* are increasing. As the total demand r increases, the optimal time at which shortages occur t_1^*, the optimal cycle length T^*, are decreases and optimal ordering quantity Q^* is increasing.

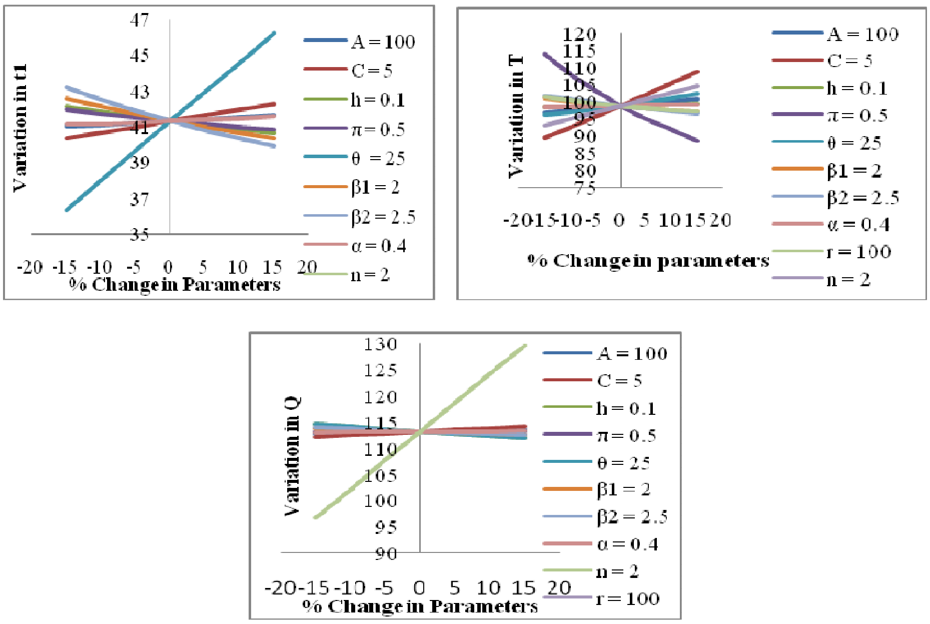


Figure 2: Relationship between Optimal Values and Parameters with Shortages

The following inferences can be made from **Table 2.** It is observed that the costs are having significant influence on the cycle length, Quantity. As the ordering cost A increases, optimal cycle length T^*, and optimal ordering quantity Q^* are increasing.

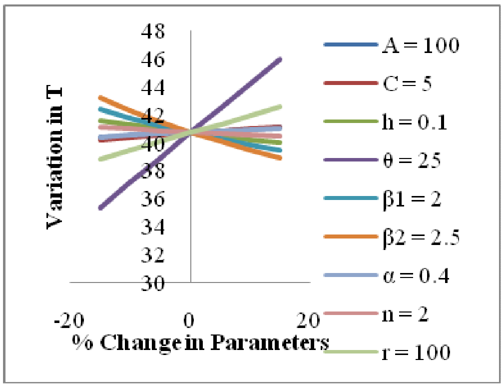
When the cost per unit C increases, optimal cycle length T^*, and optimal ordering quantity Q^* are increasing. When the holding cost h increases, optimal cycle length T^*, optimal ordering quantity Q^* are decreasing.

As the scale parameter θ increases, optimal cycle length T^*,, is increasing and optimal ordering quantity Q^*, is decreases. As the shape parameter β_1 increases optimal cycle length T^*,, and optimal ordering quantity Q^*, are decreasing. As the shape parameter β_2 increases optimal cycle length T^*,, and optimal ordering quantity Q^*, are decreasing. As the mixing parameter '□' increases optimal cycle length T^*, and optimal ordering quantity Q^*, are increasing.

As the pattern index parameter 'n' increases optimal cycle length T^*, and optimal ordering quantity Q^*, are increasing. As the total demand r increases optimal cycle length T^*, is decreasing and optimal ordering quantity Q^*, are increasing.

Table 2: Sensitivity Analysis for without Shortages

Variation Parameter	Optimal Value	% Change in Parameter						
		-15	-10	-5	0	5	10	15
A	T^*	40.3483	40.4633	40.5781	40.6927	40.8069	40.9210	41.0348
	K^*	21.4066	21.5304	21.6538	21.7768	21.8995	22.0219	22.1439
	Q^*	118.2327	118.4907	118.7497	119.0099	119.2711	119.5333	119.7966
C	T^*	40.1958	40.3722	40.5375	40.6927	40.8387	40.9763	41.1062
	K^*	19.5804	20.3132	21.0454	21.7768	22.5077	23.2380	23.9678
	Q^*	117.8933	118.2862	118.6578	119.0099	119.3438	119.6610	119.9627
h	T^*	41.5338	41.2427	40.9625	40.6927	40.4326	40.1817	39.9395
	K^*	21.0636	21.3034	21.5411	21.7768	22.0107	22.2429	22.4733
	Q^*	120.9703	120.2818	119.6292	119.0099	118.4216	117.8621	117.3295
θ	T^*	35.3038	37.1184	38.9144	40.6927	42.4542	44.1999	45.9305
	K^*	24.7808	23.6704	22.6748	21.7768	20.9626	20.2208	19.5420
	Q^*	120.9703	120.2818	119.6292	119.0099	118.4216	117.8621	117.3295
β_1	T^*	42.3860	41.7585	41.1977	40.6927	40.2347	39.8169	39.4337
	K^*	21.4318	21.5549	21.6695	21.7768	21.8776	21.9725	22.0621
	Q^*	121.0093	120.2617	119.6002	119.0099	118.4792	117.9992	117.5625
β_2	T^*	43.1702	42.2402	41.4210	40.6927	40.0400	39.4511	38.9165
	K^*	21.2289	21.4264	21.6083	21.7768	21.9334	22.0795	22.2162
	Q^*	121.6326	120.6481	119.7809	119.0099	118.3186	117.6945	117.1275
□	T^*	40.3806	40.4838	40.5878	40.6927	40.7984	40.9050	41.0125
	K^*	21.8475	21.8241	21.8005	21.7768	21.7530	21.7291	21.7051
	Q^*	118.6605	118.7763	118.8928	119.0099	119.1276	119.2459	119.3649
n	T^*	38.7881	39.4344	40.0690	40.6927	41.3058	41.9090	42.5025
	K^*	22.6918	22.3714	22.0669	21.7768	21.5001	21.2356	20.9825
	Q^*	117.3302	117.9145	118.4738	119.0099	119.5243	120.0186	120.4940
r	T^*	41.0949	40.9463	40.8130	40.6927	40.5836	40.4842	40.3933
	K^*	18.8771	19.8441	20.8107	21.7768	22.7426	23.7081	24.6734
	Q^*	101.9459	107.6325	113.3206	119.0099	124.7002	130.3915	136.0835



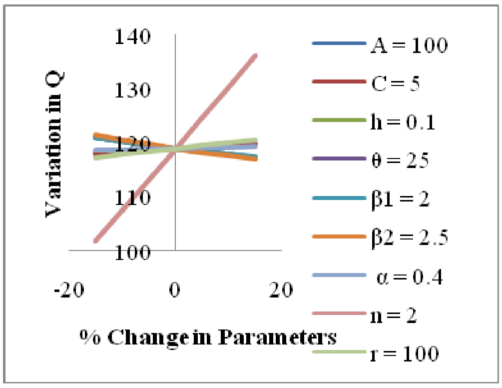


Figure 3: Relationship between Optimal Values and Parameters without Shortages

7 CONCLUSIONS

This paper addresses a novel and a new inventory model for deteriorating items with two component mixture of Pareto lifetime. Here the model analyses the inventory systems having heterogeneous commodities more close to the realistic situations. The lifetime of the commodity is characterized by two component mixture of Pareto distribution. The two component mixture of Pareto distribution includes unimodal and bimodal distributions as particular cases. The Pareto distribution gained lot of importance in reliability and life testing experiments. The two component mixture of Pareto distribution is used for the first time in modeling the inventory systems. The optimal ordering policies of the system are derived under the assumption that the demand is time dependent and follows a power pattern. The power pattern demand includes increasing, decreasing and constant rate of demand. The solution procedure of the model is demonstrated through numerical illustration using unconstrained optimization and conditions of Hessian matrix. The_sensitivity analysis of the model reveals that the lifetime distribution parameters and demand function parameters have significant influence on optimal operating policies. It is also further observed that the change in costs have significant influence on the ordering policy of the system. This model also includes some of the earlier models as particular cases. The developed model can be used for scheduling the operations at market yards, sea food industries, and chemical plants. It is possible to extend this model with other types of demand functions such as stock dependent demand, time and selling price dependent demand which will be taken up elsewhere.

REFERENCES

1. Aggarwal, S. P, Vijay, P. Goel. (1981). Inventory model with unknown demand distribution, Indian Journal of Pure Appl. Math, Vol. 12, No. 5, pp 559-565.
2. Aggarwal, S. P. (1978). A Note on an Order Level Inventory model for a system with constant rate of deterioration. OPSEARCH, Vol. 15, No. 4, pp 184-187.
3. Bhanu Priya Dash, Trailokyanath, Hadibandhu Pattnayak. (2014). An inventory model for deteriorating items with exponential declining demand and time varying holding cost, AJOR, Vol. 4, No. 1, pp 1-7.
4. Biswajit Sarkar. (2012). An EOQ model with delay in payments and stock dependent demand in the presence of imperfect production, Applied Mathematics and Computations, 218, pp 8295-8308.
5. Cochen, M. A. (1977). Joint pricing and ordering policy for exponentially decaying inventory with known demand, Nav. Res. Log. Quar 24(2), pp 257-268.

6. Covert, R. P, Philip, G.C. (1973). An EOQ model for items with Weibull distribution deterioration, AIIE. Trans 5, 323 – 326.
7. Dave. U, Shah. Y. K: A Probabilistic Inventory model for deteriorating items with load time equal to one scheduling period. European Journal Operational Research, Vol. 9,No. 3, pp 281-285 (1982)
8. Ghare, P. M, Schrader, G. F: A Model for exponentially decaying inventories, Journal of Industrial Engineering, Vol.14, 238 – 243 (1963)
9. Giri, B. C, Chaudhuri, K. S. (1997). Heuristic models for deteriorating items with shortages and time varying demand and costs, International Journal of Systems Science, Vol. 28, pp 53-59.
10. Goyal, S. K, Giri, B. C. (2001). Recent trends in modeling of deteriorating inventory', European Journal of Operational Research, Vol. 134, 1 – 16.
11. Horng Jinh Chang, Wen Feng Lin. (2010). A Partial backlogging Inventory model for Non instantaneous deteriorating items with stock dependent consumption rate under inflation, Yugoslav Journal of Operational Research, Vol. 20, No. 1, pp 35-54.
12. Jayjayanthi Ray. (2014). An inflationary inventory model with stock dependent demand and shortages, IJMTT, Vol. 10, No. 2, pp 76-84.
13. Kalpakam, S, Sapna, K. P. (1996). A lot sales (s-1, s) perishable inventory system with renewal demand, Naval Research Logistics, Vol. 43, pp 129-142.
14. Khanra. S, Buddha Dev Mandal and Biswajit Sarkar. (2013). An inventory model with time dependent demand and shortages under trade credit policy, Economic Modeling, Vol. 35, pp 349-355.
15. Nirupama Devi, K. Srinivasa Rao. K, Lakshminarayana, J. (2001). Perishable Inventory models with mixture of Weibull distribution having demand as a power function of time, Assam Statistical review, Vol. 15, No. 2, pp 70-80.
16. Nirupama Devi, K. Srinivasa Rao. K, Lakshminarayana, J. (2004). Optimal pricing and ordering policies for deteriorating inventory having mixed Weibull rate of decay, Proc. Of AP Academy of Sciences, Vol. 8, No.2, pp 125-132.
17. Pal, M. (1990). An Inventory model for deteriorating items when demand is random, Calcutta Statistics Association Bulletin, Vol. 39, pp 201-207.
18. Pentico David. W, Drake Matthew. J. (2011). An Inventory model under inflation for deteriorating items with stock dependent demand consumption rate and partial backlogging shortages. European Journal of Operational Research 168 (2), pp 463-474.
19. Philip, G. C. (1974). A Generalized EOQ model for items with Weibull distributed deterioration, AIIE Transactions, Vol. 6, pp 159-162.
20. Raafat, F. (1991). Survey of literature on continuously deteriorating inventory models, Journal of the Operational Research Society, Vol. 42, No.1, 27-37.

21. Rong, M, Mahapatra, N. K, Maiti, M. (2008). A two warehouse inventory model for a deteriorating item with partially/fully backlogged shortage and fuzzy lead time, European Journal Operational Research, Vol. 189, pp 59-75.

22. Ruxien Li, Lan. H and Mawhinney. R. J. (2010). A Review on deteriorating inventory study, Journal of Service Science and Management, Vol. 3, No. 1, pp 117-129.

23. Shah. Y. K, Jaiswal, M. C. (1977). An Order Level Inventory model for a system with constant rate of deterioration, OPSEARCH, Vol. 14, pp 174-184.

24. Srinivasa Rao, K, Srinivas, Y, Suryanarayana, B. V, Gopinath, Y. (2009).Pricing and ordering policies of an Inventory model for deteriorating items having additive exponential lifetime and selling price dependent demand rate, Indian Journal of Mathematics and Mathematical Sciences, Vol. 5, No. 1, pp 9-16.

25. Srinivasa Rao. K, Koushar Jaha Begum, Vivekananda Murthy. M. (2007). Optimal Ordering policies of inventory model for deteriorating items having generalized Pareto lifetime, Current Science, Vol. 93, No. 10, pp 1407-1411.

26. Srinivasa Rao. K, Vivekananda Murthy. M, Eswara Rao. S. (2005). Optimal ordering and pricing policies of Inventory model for deteriorating items with generalizes Pareto lifetime, Stochastic Modeling and Applications, Vol. 8, No. 1, pp 59-72.

27. Tadikamalla, P. R. (1978). An EOQ Inventory Model for items with Gamma distributed deterioration, AIIE Transactions, Vol. 10, pp 108-112.

28. Vinod Kumar Mishra. (2014). Deteriorating inventory model with controllable deterioration rate for time dependent demand and time varying holding cost, YJOR, Vol. 24, No.1, pp 87-98.

29. Vinod Kumar Mishra, Lal Saheb Singh, Rakesh Kumar. (2013). An Inventory model for deteriorating items with time dependent demand and time varying holding cost under partial backlogging, Journal of Industrial Engineering International, Vol. 9, N0. 4, pp 1-5.

30. Venkata Subbaiah, K, Srinivasa Rao. K, Satyanarayana, B. (2004). Inventory models for perishable items having demand rate dependent on stock level, OPSEARCH, Vol. 41, No. 4, pp 222-235.

31. Whitin. T. M. (1957). Theory of Inventory management, Princeton University Press, Princiton, pp 62-72.

32. Xu, X-H, Li, R-J. (2006). A two warehouse inventory model for deteriorating items with time dependent demand, Logistics Technology, No. 1, pp 37-40.